

Note: Applications of $(I+H)$ and $(I-H)$

- Let H be the normalized Walsh–Hadamard transform
- Then:

$I+H$, $I-H$

are (scaled) operators that split a vector into two orthogonal components:

$$x_+ = \frac{1}{2}(I+H)x, \quad x_- = \frac{1}{2}(I-H)x$$

- x_+ : invariant under H
 - x_- : flips sign under H
 - $x = x_+ + x_-$, and $x_+ \perp x_-$
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Applications

- **Signal decomposition**
 - Split a signal into two orthogonal components with distinct symmetry properties
- **Two-channel filtering**
 - Treat x_+ and x_- as separate bands and process independently
- **Denoising / projection**
 - Treat one component (often x_-) as noise and discard or attenuate it
- **Feature extraction**
 - Use (x_+, x_-) as a structured 2-channel representation
- **Fast subspace projection**
 - Efficiently project onto two complementary subspaces using FWHT
- **Energy diagnostics**
 - Monitor $|x_+|^2$ vs $|x_-|^2$ as a simple structural signal
- **Error / consistency detection**
 - Measure deviation from invariance via $|(I-H)x|$
- **Synchronization / alignment**
 - Use invariance and anti-invariance properties to align signals or detect offsets
- **Redundant encoding**
 - Transmit $(I+H)x$ and $(I-H)x$ for robustness and consistency checks
- **Interference separation**
 - Isolate components that behave differently under H
- **Randomized algorithms / sketching**
 - Use as structured, fast orthogonal projections

- **Iterative / splitting methods**
 - Apply different updates to x_+ and x_- in algorithms
 - **Unsupervised learning signals**
 - Use $\|(I-H)x\|$ as a symmetry-based regularizer or anomaly score
 - **Spectral probing**
 - Use the split to reveal how a vector couples to different modes under transforms
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Key intuition

- $(I+H)$ and $(I-H)$ provide a **fast, exact, orthogonal 2-way split** of any vector
 - Many applications reduce to:
process, analyze, or constrain each component differently, then recombine
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